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Analytical techniques for the estimation of mine water inflow

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Summary

This paper outlines various analytical techniques for mine water inflow estimation incorporating several refinements such as finite boundary conditions, linear, fracture and turbulent flow conditions to pumping wells and underground excavations. These modifications enable practical mining to be simulated and therefore, permit more accurate predictions of mine inflow quantities. Application of these techniques are given together with the scope of the application.

Keywords: Mine water; dewatering; well flow equations.

Introduction

In recent years the importance of mine drainage control and the need for detailed design of mine water systems at the planning stage have been highlighted (see Fernandez-Rubio, 1978; Dunn, 1982). There are several new mining projects where the sinking of shafts and the driving of drifts through the water-bearing strata have encountered major hydrogeological problems (Pocock, 1982). An accurate forecast of water inflow to mines is the basis for the design of underground drainage control installations such as pumping stations, sediment settlers, underground pumping equipment, and for finding the best operational policy throughout the life of the project. Unexpected water inflow to a shaft during sinking operations may cause considerable delay to the project due to time loss in dewatering, restoration of the flooded workings and planning and construction of new water standage facilities. The consequential cost of such unexpected flooding may be in the order of millions of pounds. An accurate prediction of the rate of inflow to mines is therefore an essential requirement for the design of large new mining projects such as the North East Leicestershire, Selby and South Warwickshire coalfields in Britain. Accurate ground water prediction is also important in existing mines working under aquifers and large bodies of water such as the undersea workings of North East England. This

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necessitates a critical review of current analytical prediction methods used in both the UK and other countries.

Scope of analytical techniques

In general terms, simple *analytical* techniques are based on estimation of the quantity of water pumped out from a well or gallery (Fig. 1) to enable the water table/piezometric surface to be lowered below the mining horizon. Simple analyses of this situation are based on steady-state conditions and transient state flow and can be modified to give an estimation of mine water

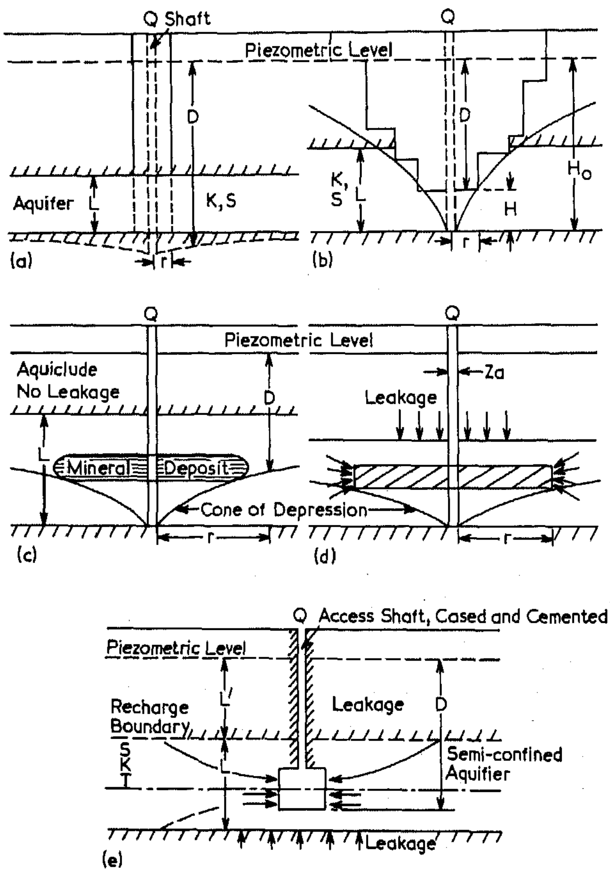


Fig. 1. Conceptual flow models forming the basis for simple analytical methods of mine water inflow estimation (see also Table 1). (a) Shaft dewatering; (b) dewatering of an open pit; (c) dewatering of an underground mine (confined aquifer); (d) dewatering of an underground mine (semiconfined aquifer); (e) dewatering of a large underground chamber (nonsteady).

inflow under idealized conditions. Examples of the well-known equations and their potential application to mine geometries are given in Table 1.

These analytical techniques are valid under the following idealized conditions: the aquifer has a seemingly infinite areal extent; the aquifer is homogeneous, isotropic and of uniform thickness over the area influenced by mining; prior to pumping, the piezometric surface and/or phreatic surface are (nearly) horizontal over the area influenced by mining; the aquifer is pumped at a constant discharge rate; the aquifer is fully penetrated by the well, and thus, water flows to the well from the entire thickness of aquifer by horizontal flow; for unsteady-state methods only the storage in the well is neglected and water removed from storage is discharged instantaneously with decline in head; and the variations of diameter of the well do not affect discharge or the draw down (linear flow).

The techniques are limited in their application to simplified flow situations because they do not take into account any of the following limiting factors: a sloping aquifer and water table; complex boundary conditions such as geometrical shape, and geological features such as faults and dykes; multiple aquifer layers such as are found typically in the Coal Measures cyclothem; variation of the storage coefficient with time; compression of the aquifer due to spatial variation of permeability with destressing and depth; inclined mineral deposits; and variable boundary conditions of overlying bodies of water such as rivers, lakes and the sea with variations in water depth.

Numerical models including finite difference and finite element models have a potential versatility for catering for all aquifer conditions and design variations. A numerical approach is often superior to an analytical model and enables a more realistic representation to be made of the interaction between ground water systems and mining. Consequently, the predicted inflow to mines and the effects of mining on the surrounding water table or potentiometric surface can be determined more reliably, provided the relevant data has been collected in a properly designed programme.

Although developments in numerical models must eventually supercede analytical models there will remain a need for a rapid and simple assessment of water inflow which can be provided by analytical models. Analytical techniques will tend to overestimate the quantity of water inflow. For example, the presence of impervious layers above the mining horizon will restrict water inflow so that it is not necessary to dewater the host rock. Certain modifications are necessary to obtain more realistic estimates, some of the situations where modifications can be applied are outlined as follows.

Finite aquifers. As the water is pumped out of a mine working below an aquifer which is finite in areal extent (bounded by faults or dykes etc. with no recharge at boundaries) at a constant rate of discharge, the radius of influence of the cone of depression will increase with time until the aquifer boundaries are reached. At this point the water table/piezometric surface of the aquifer is lowered without increasing the radius of influence of the depression cone. Consequently, the water table/piezometric surface will lower at a faster rate.

Linear flow. The basic analytical equations presume a linear flow through a porous medium. However, in mining the flow of water occurs normally through induced fractures involving high permeability coefficients and large hydraulic gradients causing turbulent flow conditions. This

Table 1. Analytical formulae for mine water inflow predictions (Theis, 1935; Jacob and Lohman, 1952; Hantush and Jacob, 1955; Hantush, 1959; Brearley, 1964; Krusman *et al.*, 1979).

(1) Theis Equation	Shaft problem (Fig. 1a)	$Q=\dfrac{4\pi KDL}{W(u)}$	(1a)
		$u=\dfrac{r^2S}{4KLt}$	(1b)
or approximate equations	Shaft problem	$Q=\dfrac{2\pi KDL}{\ln(R/r)}$	(1c)
		$R=2(Kt/2)^{\frac{1}{2}}$	(1d)
(2) Jacob–Lohman Equation	Underground mine and surface mine problems (Fig. 1b)	$Q=2\pi TDW(\lambda)$	(2a)
	Confined aquifer (Fig. 1c)	$\lambda=Tt/Sr^2$	(2b)
(3) Hantush–Jacob	Underground mine (horizontal flow) semi-confined aquifer (Fig. 1d)	$Q=\dfrac{2\pi KLD}{K_0(r/B)}$	(3a)
		$B=(KLL'/K')^{\frac{1}{2}}$	(3b)
Approximate Equation	Underground mine (horizontal flow)	$Q=\dfrac{2\pi KLD}{\ln(R/r)}$	(3c)
		$R/r=0.88+11.8(B/r)$	(3d)
		$B=(KLL'/K')^{\frac{1}{2}}$	
(4) Hantush Equation	Large underground opening (Fig. 1e) (Dudley, 1972)	$Q=2\pi TDG(\lambda, r/B)$	(4a)
		$\lambda=Tt/r^2S$	(4b)
		$r/B=r(K'/KLL')^{\frac{1}{2}}$	(4c)

B	=Leakage factor (m)$=B=(KLL'/K')^{\frac{1}{2}}$
D	=Draw down (m) to a level H from the original head H_0
$G(\lambda, r/B)$	=Hantush well function (Appendix 1)
i	=Hydraulic gradient (dimensionless)
K	=Aquifer permeability or hydraulic conductivity (m day^{-1})
K'	=Hydraulic conductivity of aquitard (m day^{-1})
$K_0(r/B)$	=Hantush–Jacob well function for a steady-state leaking aquifer (Appendix 2)
L	=Thickness of formation being dewatered (m)
L'	=Aquitard thickness (m)
Q	=Quantity of inflow (m^3 day^{-1})
R	=Effective radius of influence (m) or cone of depression at time t
R_0	=Radius of cone of depression at mine boundary
r	=Radius at which draw down is required (m)
a	=Radius of the shaft (or well) (m)
S	=Storage coefficient (dimensionless)
$T=KL$	=Transmissivity of the aquifer (m^2 day^{-1})
t	=Elapsed time (day)
u	$=r^2S/4KtL$, a variable
$W(u)$	=Theis' well function (Appendix 3)
$W(\lambda)$	=Jacob–Lohman well function (Appendix 4)

results in increasing friction losses due to turbulent flow reducing the inflow quantities. Under these circumstances nonlinear flow equations are valid.

Interfering wells. In actual mining operations efficient dewatering of an aquifer would normally require more than one pumping well resulting in interaction between the wells. This will affect such factors as radius of influence, drawdown and quantity of water to be pumped. Modified equations will be necessary to model such situations.

Dewatering by underground roadways. The essence of dewatering models is based on imaginary pumping out wells while in practice the water inflow will occur in development roadways. Consequently, the analytical methods will require modification to simulate this practical situation.

Uniform rate of inflow. The mine dewatering models discussed in this paper are strictly applicable to uniform flow conditions. With regard to inrush where the mode of flow is nonuniform and periodic, deterministic and stochastic models are applicable and outside the scope of this paper.

Some of these can be examined by looking at the analytical techniques applying to the flow of water to a well or adit which can be categorized as: linear flow in an infinite aquifer (Fig. 2); linear flow in a finite aquifer (Fig. 3); or nonlinear flow to a well in an infinite aquifer. It is also useful to examine the analytical approach to dewatering of an aquifer by interfering multiple well operations.

Linear flow to a well in an infinite aquifer

The following equation after Le'czfalvy (1982) is applicable to dewatering of an *unconfined* aquifer with a constant discharge condition

$$D = H_0 - (H_0^2 - Q/\pi K \ln R/a)^{\frac{1}{2}} \tag{5a}$$

$$R = [K(2H_0 - D)t/\mu\alpha(\ln R/a - \tfrac{1}{2}) - a^2/2(\ln R/a - \tfrac{1}{2})]^{\frac{1}{2}} \tag{5b}$$

As indicated in Fig. 2, where, in addition to nomenclature in Table 1, μ is the stress-free porosity, α is the shape factor, and H_0 the original head of water.

Conditions of validity of the equations are that there is no recharge of the aquifer and draw down is a function of time.

From Equation 5a a first approximation of draw down D is made by assuming the values of discharge rate Q and the radius of influence R for given values of the original head H, permeability coefficient K and time t. The value of draw down D together with the assumed values of R and t are substituted in Equation 5b, which is solved iteratively (usually three iterations) for R. This value of R is substituted in Equation 5a to obtain the desired value of draw down D. For routine calculations the use of a programmable calculator is desirable.

In the case of a *confined* artesian aquifer, dewatering can be performed under the following two conditions: a steady state or constant draw down condition and a nonsteady state or a constant discharge condition.

In the case of dewatering by a single artesian well, the *constant draw down* condition or *steady*

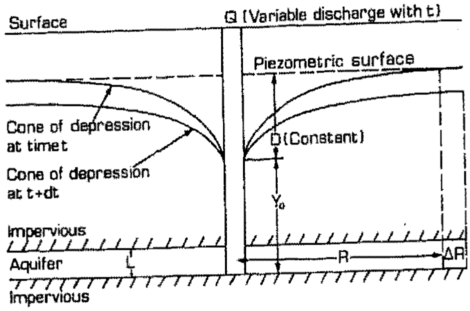


Fig. 2. Idealized conceptual model of dewatering of an infinite aquifer at constant draw down condition.

state, an equation was derived by Le'czfalvy (1982) by applying the boundary conditions to Dupuit's (1863) equation:

$$Q = 2\pi LKD/(\ln R/a) \tag{6a}$$

$$R = [(2LKt/S - a^2/2)/(\ln R/a - \tfrac{1}{2})]^{\frac{1}{2}} \tag{6b}$$

Draw down of an Artesian well in an infinite aquifer with a *constant discharge condition* or *nonsteady state* is given by (Lec'zfalvy, 1982):

$$D = (Q_t \ln R/a)/2\pi LK \tag{6c}$$

where R depends on t and its value is given by Equation 6b.

Linear flow to a finite aquifer

For mine dewatering through a borehole fully penetrating a finite aquifer with *constant discharge* condition. Le'czfalvy (1982) has derived equations for the geometry in Fig. 3. The equation presumes no recharge at the aquifer boundary and is given by

$$Q = [2\pi LKD \exp(-At/s)]/(\ln R_0/a) \tag{7a}$$

where

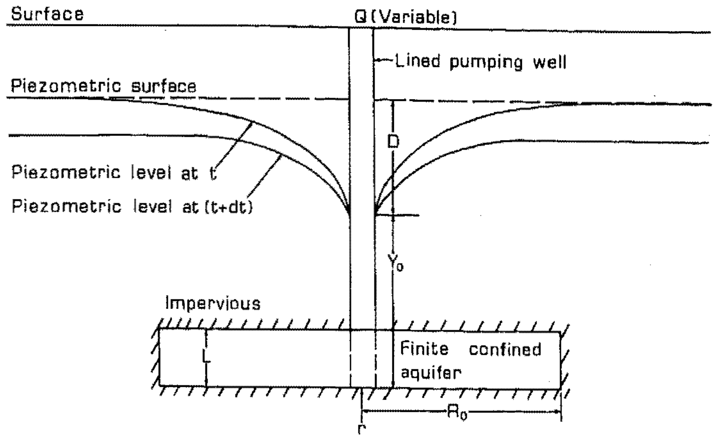
$$\text{factor } A = 2LK/(R_0^2 \ln R_0/a) \tag{7b}$$

Equation 7 is valid only after the radius of depression R has reached the mine boundary R_0. The required t_v at which the radius of depression has reached the mine boundary is given by:

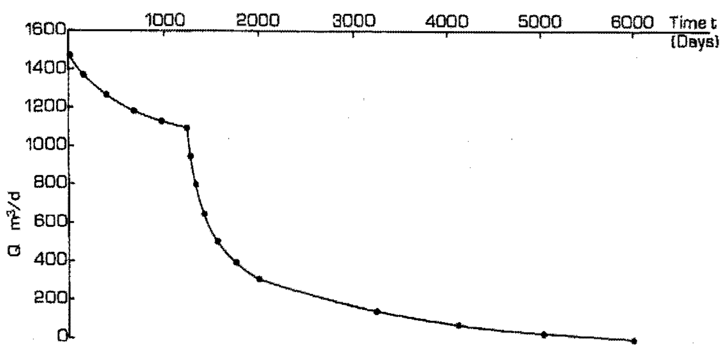
$$t_v = R_0^2(\ln R_0/a - \tfrac{1}{2})(S/2LK) + Sa^2/4LK \tag{7c}$$

The radius of influence R and the mine discharge Q for a constant discharge condition before the radius of influence reaches the mine boundary is given by Equations 6a and 6b.

For dewatering of a *finite artesian aquifer* under *constant discharge* condition, as water is pumped out at a constant rate of discharge, the radius of influence and therefore, draw down increases with time until the aquifer boundary is reached. The radius of influence and the draw



(a) Test conditions (Q-variable, D-constant)



(b) Variable discharge time curve for a constant drawdown.

Fig. 3. Dewatering of a finite aquifer at a constant draw down condition: (a) test conditions, Q variable; D constant; (b) variable discharge – time curve for a constant draw down.

down are calculated by using Equations 6b and 6c. The time t_v required to reach the aquifer boundary is calculated using 7c (see Fig. 3).

The draw down of aquifer after elapsed time t_v is then calculated using the equation:

$$D = D_0 + \frac{Q_0 t}{S R_0^2} \tag{8}$$

where D_0 is the draw down at time t_v and t is the elapse of time since pumping started but $t > t_v$.

Nonlinear flow condition to a well

The development of non-linear theory of mine water inflow can be attributed to Schmieder (1978a,b, 1979) and Pérez-Franco (1982). The analytical solution based on *unsteady flow* condition is given by:

$$D=\frac{rQ}{2\pi LgK}W(u)+\frac{C}{gK^{1/2}}\frac{Q^2}{4\pi^2L^2}\frac{R-r}{Rr}=\frac{QW(u)}{4\pi T_D}+\frac{Q^2}{4\pi^2T_T^2}\left(\frac{R-r}{Rr}\right) \tag{9a}$$

R is given by Equation 6b and $T_T=(1/2\pi)(T_D)^{1/2}$ (Schmieder, 1978a) where T_D is the linear transmissivity coefficient and r is the radius of the well (m).

The first term of Equation 9a is the Theis equation for unsteady linear flow and the second term is draw down for pure turbulent flow. Equation 9a can therefore be used to predict laminar flow by neglecting the second term, whereas for turbulent conditions, the first term can be ignored.

Similarly the steady-state flow equation is given by

$$D=\frac{Q}{2T_D}\ln\frac{R}{r}+\frac{Q^2}{4\pi^2T_T^2}\left(\frac{R-r}{Rr}\right) \qquad 9b$$

Operations of mutually interfering wells (constant discharge)

In a dewatering situation which requires the pumping of large quantities of water it may be necessary to use several pumping wells because of the limitation in capacity of individual pumps and in this situation the following quantities would be applicable (Le'czfalvy, 1982) as shown in Fig. 4.

$$D_1=\frac{Q_1}{2\pi LK}\ln\frac{R_1}{r_{10}}+\frac{Q_2}{2\pi LK}\ln\frac{R_2}{r_{10}} \tag{10a}$$

$$D_2=\frac{Q_1}{2\pi LK}\ln\frac{R_1}{b}+\frac{Q_2}{2\pi LK}\ln\frac{R_2}{r_{20}} \tag{10b}$$

where the subscripts 1, 2, 10, 20 apply to wells 1, 2, 10 and 20 and if $Q_1=Q_2=Q$, $R_1=R_2=R$.

Nonlinear flow towards an underground gallery fully penetrating a confined aquifer

The equation of nonlinear inflow of water to an underground gallery fully penetrating a *confined* aquifer (Fig. 5) under unsteady condition is given by Pérez Franco (1982):

$$D=\left(\frac{q}{LK_D}+\frac{q^2}{L^2K_T^2}\right)R_0 \tag{11a}$$

or

$$q=\frac{-\frac{1}{LK_D}+\left(\frac{1}{L^2K_D}+\frac{4R_0D}{L^2K_T^2}\right)^{1/2}}{2/L^2K_T^2} \tag{11b}$$

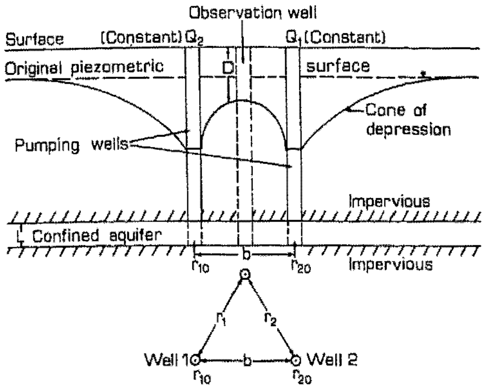


Fig. 4. Dewatering of an infinite confined aquifer by mutually interfering wells.

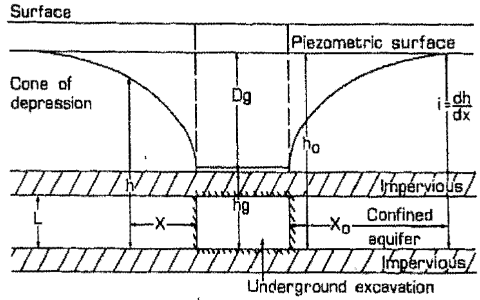


Fig. 5. Nonlinear flow to an underground excavation fully penetrating a confined aquifer.

where q is the discharge per unit length at one side of the gallery, Q is the $2\,lq$ quantity of inflow for the whole length of tunnel, h is the piezometric height at a distance x, S is the storage coefficient, t is the time when draw down is required, r is the distance measured from the face of the gallery, R is the distance from the face of trench to the place where draw down is zero (m) and is given by Equation 6, L is the aquifer thickness m, K_d is the linear hydraulic conductivity, D is the draw down at the gallery at r (m), and K_T is the turbulent hydraulic conductivity.

Nonlinear flow towards a surface mining excavation fully penetrating a saturated unconfined aquifer

The equation for nonlinear flow to a surface mining excavation has been developed by Pérez-Franco (1982):

$$q_L = \frac{K_d}{2x}(h^2 - h_g^2) \tag{12a}$$

for laminar flow and

$$q_T = K_T\left(\frac{h^3 - h_g^3}{2x}\right)^{\frac{1}{2}} \tag{12b}$$

for turbulent flow. The dewatering conditions are shown in Fig. 6.

Nonlinear flow to an open-pit excavation penetrating only the upper part of a deep aquifer

Figure 7 shows the inflow condition in an open-pit excavation situated on the upper part of a deep aquifer. The nonlinear flow equation for the above condition is based on the work of Pérez-Franco (1982):

$$y_2 - y_1 = \frac{Q}{\pi K_d} \ln \frac{x_2}{x_1} + \frac{Q^2}{\pi K_T^2} \frac{(x_2 - x_1)}{x_1 x_2} \tag{13}$$

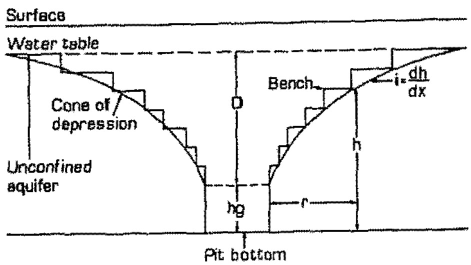


Fig. 6. Dewatering of a surface mine fully penetrating an unconfined aquifer.

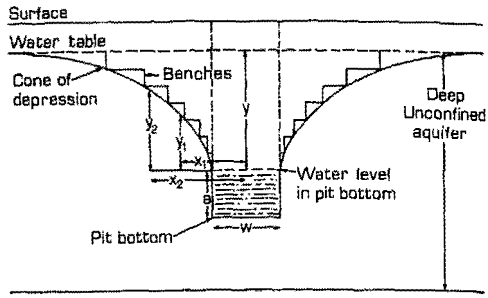


Fig. 7. Nonlinear flow to a shallow surface mine in a deep unconfined aquifer.

where x_1, x_2 are the horizontal distances from the centre of the excavation and y_1, y_2 the heights above the water level of points on the piezometric surface.

Limitations of analytical techniques

The analytical techniques for the prediction of ground water inflow are affected by several sources of inaccuracy inherent in the approach and these are as follows:

1. The idealized conceptual model to simulate the hydrological conditions may be oversimplified in geometric shape, strata section, mine and hydrological boundaries.
2. Assumptions made in the derivation of the analytical equations used may not conform with the actual field conditions and hence the calculated inflow quantities may be distorted.
3. Regional variations in the aquifer characteristics (K, S, T) cannot be easily incorporated in the analytical techniques. The most important are as follows: (1) lateral variation within the same lithological unit; (2) macroscopic changes in the aquifer characteristics with depth depending upon changes in lithology; (3) the effects of discontinuities, fractures and faults in the same lithological unit; and (4) induced mining fractures and zones of consolidations particularly in the vicinity of longwall faces. These techniques can only be applied to uniform inflow conditions and are not applicable to inrush situations.

Conclusions

The paper outlines refined analytical approaches for the prediction of mine water inflows in a finite aquifer condition and with mutually interfering wells. Nonlinear flow equations have been used to incorporate intergranular, laminar, fracture and turbulent flow conditions. This approach enables the most prominent flow condition to be simulated by neglecting the insignificant modes of inflows. This allows a more accurate prediction to be made of the likely inflow quantity. Also, in the case of nonlinear flow to a pumping well, the well diameter influences both draw down and discharge as observed in practice.

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Appendix 1. Values of the function $G(\lambda, r/B)$ – the Hantush (1959) well function calculated by Dudley (1972).

	r/B											
λ	0	1×10^{-2}	2×10^{-2}	4×10^{-2}	6×10^{-2}	8×10^{-2}	1×10^{-1}	2×10^{-1}	4×10^{-1}	6×10^{-1}	8×10^{-1}	1
1×10^{-1}	2.24	2.24	2.25	2.25	2.25	2.25	2.25	2.26	2.28	2.31	2.36	2.43
2	1.71	1.71	1.71	1.72	1.72	1.72	1.72	1.73	1.76	1.81	1.87	1.96
5	1.23	1.23	1.23	1.23	1.23	1.24	1.24	1.25	1.30	1.38	1.48	1.81
1×10^{-0}	0.983	0.983	0.984	0.985	0.986	09.87	0.900	1.01	1.07	1.18	1.32	1.49
2	0.800	0.800	0.801	0.802	0.804	0.806	0.809	0.834	0.929	1.07	1.25	1.44
5	0.632	0.628	0.629	0.630	0.633	0.637	0.642	0.682	0.824	1.01	1.22	1.43
1×10^{1}	0.534	0.534	0.535	0.537	0.541	0.547	0.554	0.661	0.793	1.01	1.22	1.43
2	0.461	0.461	0.462	0.466	0.472	0.481	0.401	0.569	0.785			
5	0.389	0.389	0.390	0.397	0.407	0.431	0.436	0.546	0.784			
1×10^{2}	0.346	0.346	0.349	0.359	0.374	0.394	0.417	0.545	0.784			
2	0.311	0.312	0.316	0.331	0.353	0.380	0.408	0.545				
5	0.274	0.276	0.284	0.309	0.341	0.374	0.406					
1×10^{3}	0.251	0.255	0.266	0.301	0.339	0.374	0.406					
2	0.232	0.238	0.255	0.299	0.330							
5	0.210	0.222	0.249	0.290								
1×10^{4}	0.196	0.216	0.248									
2	0.185	0.213	0.248									
5	0.170	0.212										
1×10^{5}	0.161	0.212										
2	0.152											
5	0.143											
1×10^{6}	0.138											

Appendix 2. Values of $K_0(r/B)$ – the Hantush–Jacob (1955) well function.

N	$r/B = N \times 10^{-2}$	$N \times 10^{-5}$	$N \times 10^{-3}$	N
1.0	7.0237	4.7212	2.4271	0.4210
1.5	6.6182	4.3159	2.0300	0.2138
2.0	6.3305	4.0285	1.7527	0.1139
2.5	6.1074	3.8056	1.5415	0.0623
3.0	5.9251	3.6235	1.3725	0.0347
3.5	5.7709	3.4697	1.2327	0.0196
4.0	5.6374	3.3365	1.1145	0.0112
4.5	5.5196	3.2192	1.0129	0.0064
5.0	5.4143	3.1142	0.9244	0.0037
5.5	5.3190	3.0195	0.8466	
6.0	5.2320	2.9329	0.7775	0.0012
6.5	5.1520	2.8534	0.7159	
7.0	5.0779	2.7798	0.6605	0.0004
7.5	5.0089	2.7114	0.6106	
8.0	4.9443	2.6475	0.5653	
8.5	4.8837	2.5875	0.5242	
9.0	4.8266	2.5310	0.4867	
9.5	4.7725	2.4776	0.4524	

Appendix 3. Theis well function $W(u)$ after Theis (1935).

	u or U_{xy}							
N	$N \times 10^{-14}$	$N \times 10^{-12}$	$N \times 10^{-10}$	$N \times 10^{-8}$	$N \times 10^{-6}$	$N \times 10^{-4}$	$N \times 10^{-2}$	N
1.0	31.6590	27.0538	22.4486	17.8435	13.2383	8.6332	4.0379	0.2194
2.0	30.9658	26.3607	21.7555	17.1503	12.5451	7.9402	3.3547	0.04890
3.0	30.5604	25.9552	21.3500	16.7449	12.1397	7.5348	2.9591	0.01305
4.0	30.2727	25.6675	21.0623	16.4572	11.8520	7.2472	2.6813	0.003779
5.0	30.0495	25.4444	20.8392	16.2340	11.6280	7.0242	2.4679	0.001148
6.0	29.8672	25.2620	20.6569	16.0517	11.4465	6.8420	2.2953	0.0003601
7.0	29.7131	25.1079	20.5027	15.8976	11.2924	6.6879	2.1508	0.0001155
8.0	29.5795	24.9744	20.3692	15.7640	11.1589	6.5545	2.0269	0.0000376
9.0	29.4618	24.8566	20.2514	15.6462	11.0411	6.4368	1.9187	0.0000124

Appendix 4. Jacob–Lohman well function $W(\lambda)$ after Jacob and Lohman (1952).

	λ								
N	$N \times 10^{-4}$	$N \times 10^{-2}$	N	$N \times 10^{2}$	$N \times 10^{4}$	$N \times 10^{6}$	$N \times 10^{8}$	$N \times 10^{10}$	$N \times 10^{12}$
1	56.9	6.13	0.985	0.346	0.1964	0.1360	0.1037	0.0838	0.0764
2	40.4	4.47	0.803	0.311	0.1841	0.1299	0.1002	0.0814	0.0744
3	33.1	3.74	0.719	0.294	0.1777	0.1266	0.0982	0.0801	0.0733
4	28.7	3.30	0.667	0.283	0.1733	0.1244	0.0968	0.0792	0.0726
5	25.7	3.00	0.630	0.274	0.1701	0.1227	0.0958	0.0785	0.0720
6	23.5	2.78	0.602	0.268	0.1675	0.1213	0.0950	0.0779	0.0716
7	21.8	2.60	0.580	0.263	0.1654	0.1202	0.0943	0.0774	0.0712
8	20.4	2.46	0.562	0.258	0.1636	0.1192	0.0937	0.0770	0.0709
9	19.3	2.35	0.547	0.254	0.1621	0.1184	0.0932	0.0767	0.0706
10	18.3	2.25	0.534	0.251	0.1608	0.1177	0.0927	0.0764	0.0704